Maxwell, and the Mathematics of Metaphor

by Tasneem Zehra Husain

It is practically a rite of passage for physics majors. We study Maxwells equations - the illuminating set of relationships that reveal the nature of light; we marvel at the power and grace of this compact quartet, and can't resist a chuckle when - inevitably - we come across the t-shirt that says "God said [Maxwell's Equations] And there was light." Something about that sticks. We remember the t-shirt years later, even if we can't write down the equations anymore.

But, even though James Clerk Maxwell could boast several outstanding accomplishments - including taking the first ever color photograph, and unleashing a fictional demon that outwits entropy - for far too many of us, our association with this brilliant scientist begins and ends with the famous equations that govern electromagnetic radiation.

There is no cult of genius surrounding Maxwell. Unlike Einstein, Feynman or more recently, Hawking, Maxwell has no groupies; his quotes don't adorn bumper stickers, physics students don't own collections of his lectures or writing, and I don't know of anyone (save Einstein) who put up a poster of Maxwell in their workspace.

Like thousands of other physics students who went to college, studied Maxwell's equations, and bought the t-shirt, I felt no real bond with the man until some years ago, when a writing project (to which I shall forever remain indebted) led me to find out more about him.

I read Maxwell's writings as part of my research, and it was love at first letter. I was completely enchanted by the mind revealed in, and between, the lines; it was an investigative, creative, whimsical creature, with scintillating wit and lyrical expression. Over the months, as I read more, my initial intellectual infatuation developed into a deep fondness and a genuine respect. I found the flow of Maxwell's logic and the dance of his ideas simply beautiful. I read and re-read his words for the pleasure of having them stream through my mind, but also in the hope that if they performed his choreography enough times, my thoughts might learn

to move that way on their own.

Casting around today for a piece of writing that might serve as an introduction to Maxwell, I settled upon his 1870 address to the members of the British Association (whom Maxwell teasingly referred to as the British 'Asses') about scientific metaphor. This is not an idea that is articulated very often, especially not this clearly, but it is precisely the sort of overarching theme that I think should be emphasized in physics classrooms everywhere.

In his typical, beautiful manner, Maxwell begins by praising the "*penetrating insight*" that led to "*that sanctuary of minuteness and of power where molecules obey the laws of their existence, clash together in fierce collision, or grapple in yet more fierce embrace, building up in secret the forms of visible things*". But even with this success to bolster us, how, he asks, are we to enter that "*still more hidden and dimmer region where Thought weds Fact, where the mental operation of the mathematician and the physical action of the molecules are seen in their true relation?*"

He reminds us that the task of mathematicians is to "*perform certain mental operations on the symbols of number or of quantity*". These operations range from simple to complex, and enable us to express the same thing in many different forms.

As a trivial example, consider the number 2. This can be re-written in ways as various as

723 - 721

56 x 1/28

or the infinite sum 1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + (and so on, with each successive term being half the previous one)

While the first two expressions are easily discernible as the number 2, the third might not seem quite so obvious.

This is the sort of thing Maxwell refers to when he says "*The equivalence of these different forms, though a necessary consequence of self-evident axioms, is not always, to our minds, self-evident; but the mathematician, who by long practice has acquired a familiarity with many of these forms, and has become expert in the processes which lead from one to another, can often transform a perplexing expression into another which explains its meaning in more intelligible language.*"

The task of physicists, on the other hand, is to deduce the physical laws that govern natural phenomena. We accomplish this by observing a particular phenomenon under a wide range of circumstances, varying what parameters are possible, and drawing what conclusions we can from the resulting data.

A simple illustration is provided by the famous experiment in which two balls of varying weight, thrown down from a tall tower, reach the ground at the same time. Since that might seem somewhat counter intuitive (most people who have not studied physics, even today, expect the heavier ball to fall to the ground faster) we investigate the phenomena further, tweaking the set up and studying how these changes affect the outcome. We could vary the weights of the balls, perhaps even their sizes and/or the materials from which they are made. We could vary the height from which they are thrown down - first floor, tenth floor, all the way at the top; and when, every single time, we find that both balls hit the ground simultaneously we conclude that the law governing this descent - the law of gravitation - must operate in such a way that it is blind to the mass of the falling object.

Having thus described the domains in which mathematicians and physicists operate, Maxwell goes on to say that the student who is acquainted with several different sciences, often finds himself at an advantage, since *"the mathematical processes and trains of reasoning in one science resemble those in another so much that his knowledge of the one science may be made a most useful help in the study of the other. When he examines into the reason of this, he finds that in the two sciences he has been dealing with systems of quantities, in which the mathematical forms of the relations of the quantities are the same in both systems, though the physical nature of the quantities may be utterly different."*

And so, Maxwell plants the seeds for us to recognize a likeness of a different kind, *"a classification of quantities on a new principle, according to which the physical nature of the quantity is subordinated to its mathematical form."* This new sort of similarity is not physically apparent, but is instead a structural resemblance between the laws that govern two systems.
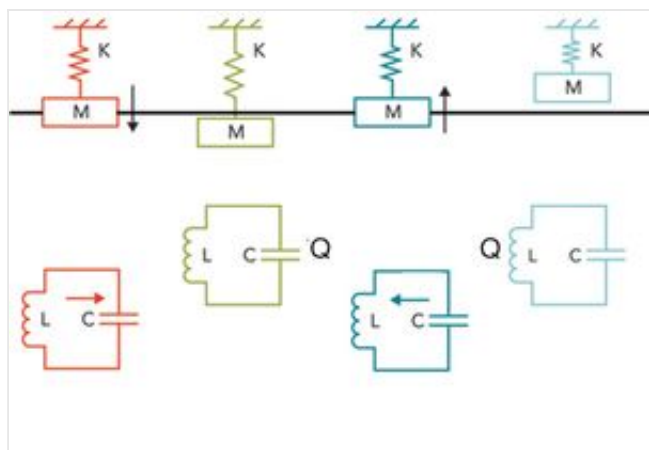
As an example, consider a mass tied to a spring, and a circuit with an inductor and a capacitor. On the surface, these systems look vastly different, but they turn out to be intrinsically connected.

Think of what happens when the spring is pulled, and then released; the mass begins to oscillate about its initial rest position. The stretched spring tends to compress, pulling the mass upward, but when it has been squeezed by a certain amount, the spring bounces back, pushing the mass downward; and so it continues - in an ideal set-up (frictionless, free of air-resistance, etc.), it would do so forever. The circuit, on the other hand, does not physically move at all. The relationship is more subtle than that.

It turns out that if the capacitor is fully charged, and the switch then closed (this is analogous to the spring

being pulled to its limit, and let go), the capacitor immediately begins to discharge, and a current flows in the circuit, charging the inductor; when the capacitor loses all charge, the inductor is fully charged; once this limit is reached, the inductor begins to discharge, transferring charge to the capacitor (in the opposite polarity to before, for those of an electrical bent).

The equations that govern the oscillating charge in this circuit are identical in form to those that describe the oscillating position of the mass on the spring. The physical manifestations are vastly different - one system is mechanical, while the other is electric - but the relationships between the physical quantities is identical.

A truly scientific illustration, says Maxwell, *"is a method to enable the mind to grasp some conception or law in one branch of science, by placing before it a conception or a law in a different branch of science, and directing the mind to lay hold of that mathematical form which is common to the corresponding ideas in the two sciences, leaving out of account for the present the difference between the physical nature of the real phenomena.*

The correctness of such an illustration depends on whether the two systems of ideas which are compared together are really analogous in form, or whether, in other words, the corresponding physical quantities really belong to the same mathematical class. When this condition is fulfilled, the illustration is not only convenient for teaching science in a pleasant and easy manner, but the recognition of the formal analogy between the two systems of ideas leads to a knowledge of both, more profound than could be obtained by studying each system separately."

"*Physical research is continually revealing to us new features of natural processes,*" Maxwell says, "*and we are thus compelled to search for new forms of thought appropriate to these features*". It would thus be both prudent and efficient, to see if "*the ideas derived from one department of physics may be safely used in forming ideas to be employed in a new department*" and if so, under what conditions.

Maxwell bestows the term "**Scientific Metaphor**" upon the "*figure of speech or of thought by which we transfer the language and ideas of a familiar science to one with which we are less acquainted*".

Perhaps he held the example of the spring and circuit in mind, when he wrote "*There are certain electrical phenomena, again, which are connected together by relations of the same form as those which connect dynamical phenomena. To apply to these the phrases of dynamics with proper distinctions and provisional reservations is an example of a metaphor of a bolder kind; but it is a legitimate metaphor if it conveys a true idea of the electrical relations to those who have been already trained in dynamics.*"
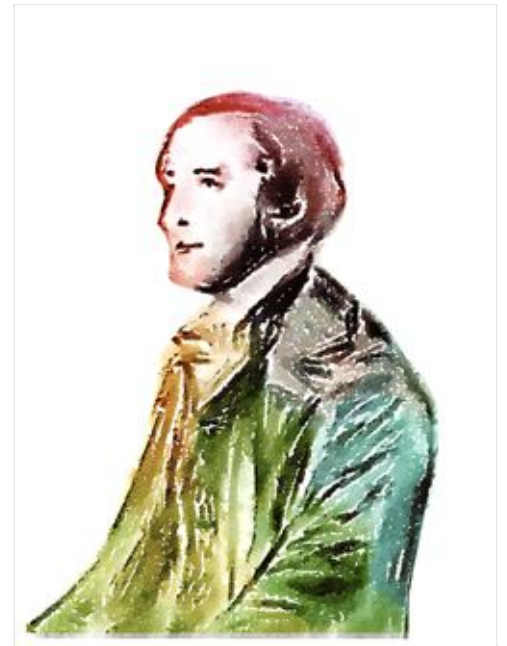
He elaborates: "*These generalized forms of elementary ideas may be called metaphorical terms in the sense in which every abstract term is metaphorical. The characteristic of a truly scientific system of metaphors is that each term in its metaphorical use retains all the formal relations to the other terms of the system which it had in its original use.*" And, in what is perhaps my favorite sentence in this entire address, Maxwell says: "*The method is then truly scientific—that is, not only a legitimate product of science, but capable of generating science in its turn.*"

But where the method is worthy of being dubbed scientific, I can't help deeming the phenomenon poetic. New science being born from structural similes and mathematical metaphors. What a lovely thought.

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