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Emmy Noether: Poet of Logical Ideas

by Tasneem Zehra Husain

"Pure mathematics is, in its way, the poetry of logical ideas". - Albert Einstein, (Obituary forEmmy Nother)

The first time you encounter a truly dazzling idea, its light seems almost blinding; slowly, your eyes grow more accustomed, and the glare dulls down to a glow which pleasantly illuminates your outlook. At least, that's how it usually happens, but Noether's Theorem is in a class of its own. I first came across it as a graduate student, about fifteen years ago, and to this day, I am stunned by its unfading brilliance.

I find it a travesty that Emmy Noether and her beautiful work are not more widely known. With International Women's Day just behind us, and Noether's birth anniversary around the corner, this seems like a good time to right that wrong. But before I introduce you to Emmy Noether, let me first tell you about her work. That, I am convinced, is how she would have wanted it.

Symmetries & Conservation Laws

The fact that the total energy of a system must always stay fixed, has been used to tremendous effect, countless times and in several contexts; from calculating the height to which a ball will rise, to predicting the existence of the neutrino. The laws of conservation of energy, momentum and charge, have long been considered sacrosanct, but for centuries, no one knew where they came from. To what do we owe the pleasure of their (very welcome) protection? Emmy Noether figured it out: Conservation laws arise out of symmetries, she said. And suddenly, just like that, there was a deeper underlying reason behind these mysteriously powerful statements - they had an origin.

Noether's theorem states that, to every (continuous) symmetry of a theory, there corresponds a conservation law.

What exactly does that mean? As physicist Philip Morrison writes, "symmetry is related to the indiscernibility of differences. Once you walk into the hall of a Palladian building, you can't quite remember whether you turned left or right". ‘The indiscernibility of differences', while not a formal definition, is a good working description. An object is symmetric if some operation can be performed on it without leaving a trace: a circle, for instance, is rotationally symmetric, because can be rotated (about its center) through any arbitrary angle, and no one will know the difference.

The symmetries Noether was concerned with are far more abstract - they are symmetries of the equations that describe a process (more properly speaking, symmetries of the Lagrangian), not symmetries of the objects participating in the process. In other words, they refer to differences that are indiscernable to the laws of physics.

That is not nearly as abstruse at it sounds. We function on the implicit assumption that the laws of physics will stay constant from one moment to the next. If I drop an apple from a certain height today, I expect it to fall to the ground in exactly the same time again, if I repeat the experiment again two weeks later. The apple itself will definitely have changed in this time. It might well have rotted. But the laws of classical mechanics, which dictate how the apple falls, have not changed.

Another familiar assumption we make is that the laws of physics are independent of position. If an apple is from the same height, in China and in Canada, it should fall to the ground in the same time. Because, while Canada can obviously be visually discerned from China - the *laws of physics* are the same in both places.

These observations might seem too trivial to state, but as Noether showed, they give birth to the deep conservation laws we depend on. Energy conservation is a direct consequence of the constancy of the laws of physics through time. Momentum conservation follows from the fact that these laws stay the same across space.

The conservation of energy and momentum are convenient examples because they are familiar, and the corresponding symmetries are easily visualizable, but Noether's theorem is far more powerful than this. It applies to any (continuous) mathematical transformation that leaves the equations of a system invariant. The operation in question need not necessarily correspond to a quantity we can identify with physically; the conservation of electric charge, for instance, follows from a phase symmetry in the equations of quantum electrodynamics.

And that is only the beginning. Noether's theorem opens the door to a whole new universe of possibilities: we can formulate theories to have certain symmetries built-in, and the corresponding conversation laws will automatically be imposed as a consequence; any time we identify a symmetry of a system, we are assured that a conservation law exits as well; conversely, if a quantity appears to be conserved, we set out to unearth the hidden symmetry. The elegance of Noether's theorem truly is unparalleled. I don't know that we will ever fathom its depths. But even though her theorem revolutionized the way physics was formulated, in her own mind, Noether was a mathematician first and foremost.

The Mother of Abstract Algebra.

Algebra focuses on general relationships, instead of specifics. It liberates us from the necessity of endless enumeration, and allows us to make one overarching statement. Instead of saying "Since 2 is an even number, 4 is also. If 4 is an even number, 6 is also. If 6 is an ...." ad infinitum, one can simply say "If n is an even number, n + 2 is also," and be done. This statement holds, regardless of the precise value of n. It is a universal truth.

The ability to extract global statements from a plethora of particulars is extremely useful. It lifts you above the mundane, to a vantage point from where you can think in far broader strokes. Noether had this gift in spades. She pioneered, and excelled in, the field of abstract algebra, which involves the generalization not only of numbers, but the *operations* performed upon the numbers, (addition, subtraction, multiplication etc). In abstract algebra, one goes to the heart of addition, to see it merely as an operation performed between two numbers (no matter how many numbers we add, we always add two at a time), such that the result does not depend on the order in which the numbers are placed. Similar abstractions can be made for multiplication, and all sorts of elaborate mathematical structures can be created as a result. These groups, and rings and fields, as they are called, exhibit all manner of interesting relationships and interdependencies, which are true, regardless of the context.

Noether's abstract algebra enabled her to get a tangible grip on objects which were unfamiliar. Since the formulation was free of visual representation, and drew only on logical relationships, it could be used to probe otherwise inaccessible spheres of thought. Her approach to mathematics was surmised in the maxim she often repeated "All relations among numbers, functions and operations become clear, generalizable, and truly fruitful only when they are separated from their particular objects and reduced to general concepts."

Noether's new and revolutionary techniques left even Einstein in her debt. In the special theory of relativity, where space and time warped and changed, there were certain privileged quantities that were left invariant. Grabbing on to these, Noether was able to provide a much needed mathematical formulation for relativity.

Einstein called Noether "the most significant creative mathematical genius thus far produced since the higher education of women began," and he was far from alone in this assessment.

The Origin of Coordinates

Amalie Emmy Noether was born on March 23, 1882 in Erlangen, Germany. She was the oldest child of four, and the only daughter of distinguished mathematics professor Max Noether, and his wife Amalia, but Emmy's mathematical genius could not be attributed to her genes. As her colleague Edmund Landau observed: "Max Noether was the father of Emmy Noether. Emmy is the origin of coordinates in the Noether family."

As a girl, Emmy went to grammar school. She learnt to play the piano, enjoyed dancing, and obtained a certification to teach French and English at school, but instead of sticking to a conventional career, Emmy opted to study mathematics. At the time, it was only possible for women to study at German universities as auditors, and that too, if they received approval from the professors. Despite not being able to officially enroll in classes, Emmy passed the doctoral examination and became a PhD candidate - the only female in her class of 47. At Göttingen, then "the Mecca of Mathematics," Emmy attended lectures by leading mathematicians such as David Hilbert, Felix Klein and Herman Minkowski. Paul Gordon, "the king of invariant theory" was her thesis advisor, and she received her doctorate *summa cum laude* in 1907.

It was unheard of for women to find a university position, so Emmy assisted her father at Erlangen instead. She was allowed to lecture and teach, but the position was unofficial and came with no pay. The constraints of bureaucracy could not, however, keep her reputation from growing. In 1915, Emmy published the theorem which linked conservation laws to symmetries and physicists were quick to pay intellectual tribute.

Hilbert and Klein invited Emmy to join them at Göttingen. They lobbied to have her become an official member of the faculty, but to no avail. The University voiced its objection thus: "What will our soldiers think when they return to the university and find they are expected to learn at the foot of a woman?" Hilbert fought back, saying he did not see how her gender mattered "After all, we are a university and ... not a bathing establishment," but his request was denied. Einstein too, was thoroughly disappointed at this outcome. The university finally relented in 1919, and at long last, Emmy won the right to call herself a

Privatedozent - a non-tenured, virtually unpaid, lecturer.

In 1922, she became an Associate Professor. Though still not tenured, she could at least teach under her own name, hold examinations, and direct dissertations. Emmy became *Doktormutter* to nine PhD students, all of whom worked in the radical new field she had pioneered. This kind of close mentoring, the nourishing of gifted minds, was her strength. In classroom settings, Emmy did not exactly shine. Her lectures were not thought out, she planned as she spoke, quite fast, often interrupting herself as new ideas came to mind. But, with a small, close-knit group of bright students, who did the work necessary to keep up with her, Emmy really came into her own.

She was referred to as "*Der Noether*," (the masculine form being used as a token of respect) and the young men and women who gathered around her came to be called "the Noether Boys". While she might not have been the most methodical teacher, Emmy was one of the most passionate. Her commitment, both to her subject and her students, shone through. Her 'boys' flocked around her, "like a clutch of ducklings about a kind, motherly hen," said Jakob Levitski, who was one of them.

Emmy was careless of her appearance. Her hair fell out of her pins, and her dress was somewhat disheveled. She was not overtly feminine in her manner, yet she was extremely maternal. As the Russian mathematician Paul said "her feminine psyche came through in the gentle and delicate lyricism that lay at the foundation of the wide-ranging but never superficial relationships connecting her with people, with her avocation, with the interests of all mankind. She loved people, science, life with all the warmth, all the joy, all the selflessness and all the tenderness of which a deeply feeling heart—a woman 's heart—was capable."

Emmy was charmingly unselfconscious and very good fun. She took her classes on long walks, held lectures under the trees, and was always up for a laugh. In keeping with Göttingen tradition, any doctoral candidate passing the oral examination must climb the bronze fountain in front of the city hall and kiss the goose girl. Quite obviously, this did not have official sanction, and most professors simply turned a blind eye. Noether, who indulged and enjoyed these high spirited antics, took her students to the fountain herself, cheering while they climbed up to steal the well-earned kiss.

Emmy nurtured her students with great patience and understanding. She nourished them and challenged them, carefully training them how to think with clarity. She put up with their incessant questions, and their perpetual presence. Emmy welcomed them to her apartment, and fed them (sometimes burnt) vanilla pudding, in mismatched cups and bowls, while she spoke to them of mathematics, frequently quoting Laplace: "If the human mind were limited to the accumulation of facts, science would be nothing more than a list. It would show us nothing of the laws of nature."

The scientific salon that grew around Emmy was vibrant, exciting, and informal. Passionate discussions were held, as scholars of all levels - from visiting dignitaries to students - sat on miscellaneous furniture pieces, or even the floor, of Emmy's welcoming apartment. Old concepts were framed in new ways, and radical new ideas were discussed. Everyone spoke with abandon, and - in a complete break from the academic tradition of guarding one's precious research - Emmy shared her emerging ideas with everyone, scattering them around generously, for any student to take, grow, and own.

For a woman who had been cheated all her life of the accolades and position that was rightfully hers, Emmy remained completely un-embittered and cheerfully unconcerned with keeping the glory for herself. She was interested in the development of the mathematical ideas first and foremost, and then in the development of these brilliant minds that had been entrusted to her care. Even though she personally published only 37 research papers, her influence over the field spread through each of her students, several of whom went on to become great mathematicians. She truly deserves to be called the ‘mother' of abstract algebra.

In 1933, while she was at her prime, the Nazis forced Emmy to leave Göttingen. She had many friends and admirers who lobbied on her behalf, and was quickly offered a professorship at Bryn Mawr College in the United States. Here, for the first time, at over fifty years of age, Emmy had a paid, faculty position, and female colleagues.

Speaking of Emmy's reaction at being forced into exile, Hermann Weyl later said "Her courage, her frankness, her unconcern about her own fate, her conciliatory spirit, were, in the midst of all the hatred and meanness, despair and sorrow, a moral solace." Perhaps Emmy's gift for abstraction extended to realms other than algebra. Perhaps she did not hold grudges because she was able to rise above the particulars of her individual life, to see the larger picture.

When she died in 1935, from a postoperative infection, Einstein penned a moving obituary. Where most of us are consumed in the day to day struggle to improve our worldly lot, he wrote "there is, fortunately, a minority composed of those who recognize early in their lives that the most beautiful and satisfying experiences open to humankind ... are bound up with the development of the individual's own feeling, thinking and acting. The genuine artists, investigators and thinkers have always been persons of this kind. However inconspicuously the life of these individuals runs its course, none the less the fruits of their endeavors are the most valuable contributions which one generation can make to its successors."

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